SLENDER COLUMN INTERACTION DIAGRAMS

by Eldon F. Mockry and David Darwin

A procedure for developing slender column interaction diagrams is presented. The provisions of ACI 318-77 for the approximate evaluation of slenderness effects are utilized. Conventional interaction diagrams are modified using a nondimensional parameter characterizing column slenderness. The interaction diagrams developed provide a direct solution for the reinforcing ratio of single columns. The approach presented will reduce the design time for reinforced concrete structures.

Keywords: buckling; **columns (supports)**; frames; loads (forces); long columns; moments; **reinforced concrete; slenderness ratio;** structural design; structural engineering.

The task of accounting for slenderness effects in reinforced concrete columns can be a tedious experience. The designer may have difficulty in making initial assumptions, which may lead to inefficient iteration. Therefore, a design procedure leading to a direct solution is desirable. The purpose of this paper is to present a method for constructing slender reinforced concrete column interaction diagrams in accordance with ACI 318-77.[1] The procedures are presented in greater detail in Reference 2.

CODE EQUATIONS

By way of introduction, the approximate slenderness provisions of ACI 318-77 are summarized below:

$$M_c = \delta M_2 \quad \text{ACI Eq. (10-6)} \qquad (1)$$

in which M_c is the "factored moment to be used for design of compression member;" M_2 is the "value of larger factored end moment on compression member calculated by conventional elastic frame analysis, always positive;" and δ is the "moment magnification factor."

$$\delta = \frac{C_m}{1 - (P_u/\phi P_c)} \quad \text{ACI Eq. (10-7)} \qquad (2)$$

in which C_m is "a factor relating actual moment diagram to an equivalent uniform moment diagram;" P_c is the "critical load;" P_u is the "factored axial load at given eccentricity $\leqslant \phi P_n$;" P_n is the "nominal axial load strength at given eccentricity;" and ϕ is the "strength reduction factor."

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \quad \text{ACI Eq. (10-8)} \qquad (3)$$

in which EI is the "flexural stiffness of compression member;" k is the "effective length factor for compression members;" and ℓ_u is the "unsupported length of compression member."

$$EI = \frac{(E_c I_g/5) + E_s I_{se}}{1 + \beta_d} \quad \text{ACI Eq. (10-9)} \qquad (4)$$

or conservatively

$$EI = \frac{E_c I_g/2.5}{1 + \beta_d} \quad \text{ACI Eq. (10-10)} \qquad (5)$$

in which E_c is the "modulus of elasticity of concrete;" E_s is the "modulus of elasticity of reinforcement;" I_g is the "moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement;" I_{se} is the "moment of inertia of reinforcement about centroidal axis of member cross section;" and β_d is the "ratio of maximum factored dead load moment to maximum factored total load moment, always positive."

Section 10.11.5.3 of ACI 318-77 states, "In Eq. (10-7), for members braced against sidesway and without transverse loads between supports C_m may be taken as

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \quad \text{ACI Eq. (10-11)} \tag{6}$$

but not less than 0.4. For all other cases, C_m shall be taken as 1.0." M_1 is the "value of smaller factored end moment on compression member calculated by conventional elastic frame analysis, positive if member is bent in single curvature, negative if bent in double curvature;" and M_2 is the "value of larger factored end moment on compression member calculated by conventional elastic frame analysis, always positive."

For unbraced frames, Section 10.11.6.2 states, "In Eq. (10-7), P_u and P_c shall be replaced by the summations ΣP_u and ΣP_c for all columns in a story."

GENERALIZED INTERACTION DIAGRAMS

The development of slender reinforced concrete column design aids involves the incorporation of slenderness effects into the column interaction diagrams. The most useful design aids allow the interaction diagrams to be expressed in terms of the axial load and the (unmagnified) moment obtained from an elastic frame analysis. Design aids of this type can be constructed in a straightforward manner, because slenderness effects are manifested in the form of a magnified moment, and the degree of magnification is a function of axial load and geometry, which are known for each point on an interaction diagram.

Conventional interaction diagrams[3] generalize the axial and bending capacities in terms of stress to negate the requirement for individual interaction diagrams for specific cross sections. The generalized capacities, the axial load index and the moment index, are $\phi P_n/A_g$ and $\phi M_n/A_g h$, respectively. This generalized approach is also desirable for slender column interaction diagrams. To do this, the slenderness effects must be expressed in terms consistent with the generalized capacities. Since conventional interaction diagrams are plotted for specific reinforcing ratios for each column configuration, in this development, expressions for the moment magnifier are also written as functions of the reinforcing ratio and the column configuration, as well as the column slenderness. By dividing the bending capacity of a section by the corresponding moment magnifier, axial load index-moment index relations are used to produce interaction diagrams expressed in terms of the axial load index, $\phi P_n/A_g$, and an index reflecting the unmagnified moment, $C_m M_2/A_g h$.

Expressions for the critical load and the corresponding moment magnifier, written consistently with the axial load and moment indices, will now be developed.

Development of the Technique

The value of EI in ACI Eq. (10-8) may be taken as the larger of the two values given by ACI Eq. (10-9) and (10-10). Selecting the larger value for EI results in a larger critical load, giving a more economical design. ACI Eq. (10-9) is dependent on the column reinforcement. When this equation is used for design, the conventional approach is to 1. assume a reinforcing ratio, 2. calculate the critical load, 3. calculate the moment magnifier, 4. magnify the moments, 5. select the reinforcing ratio from the conventional interaction diagrams, 6. compare the reinforcing ratio with the assumed value, and 7. revise, if necessary, by repeating steps 1 through 6. To eliminate the need for assuming an initial reinforcing ratio and the subsequent revisions to the solution, the moment magnifier, δ (which depends upon P_c, and thus the reinforcing ratio), is incorporated into the interaction diagrams.

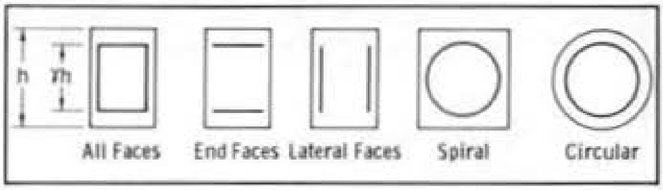

Fig. 1 — Column and reinforcing configurations.

To express the critical load in terms consistent with the axial index, the flexural stiffness, EI, must be "generalized." The contribution of the reinforcing steel to the moment of inertia, I_{se}, may be written as

$$I_{se} = C_{se} \rho A_g \gamma^2 h^2 \tag{7}$$

in which C_{se} is a characteristic coefficient for each reinforcing configuration, ρ is the total reinforcing ratio $= A_{st}/A_g$; γ is the ratio of the distance between the centroids of the reinforcement in opposite faces parallel to the neutral axis of bending to the column depth, h, as shown in Fig. 1, and is called the geometric index; A_g is the gross area of the cross section; and h is the depth of the cross section (i.e., the dimension perpendicular to the neutral axis of bending). For different reinforcing configurations, C_{se} may be shown to be:[3]

1/6 for rectangular columns with equal reinforcement distributed in each face as a thin plate,

1/4 for rectangular columns with equal reinforcement distributed in the end faces,

1/12 for rectangular columns with equal reinforcement distributed in each lateral face as a thin plate, and

1/8 for rectangular or circular columns with the reinforcement distributed as a uniform cylinder.

The moment of inertia of the gross cross section, I_g, may be expressed as

$$I_g = C_g A_g h^2 \tag{8}$$

in which C_g is a characteristic coefficient based on the shape of the gross cross section. For rectangular and circular gross cross sections, C_g is 1/12 and 1/16, respectively.

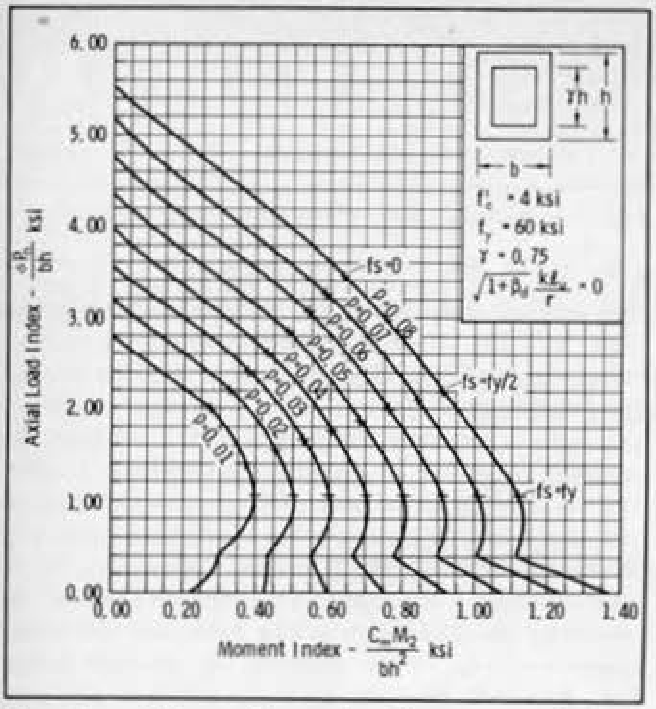

Fig. 2 — Column interaction diagram — Without slenderness effects (1 ksi = 6.9 MPa).

Using ACI Eq. (10-9), EI may be written

$$EI = \frac{\alpha A_g h^2}{1+\beta_d} \tag{9}$$

in which $\alpha = \frac{E_c C_r}{5} + E_s C_m \varrho \gamma^2$

The critical load is generalized as the critical stress, P_c/A_g, which can be written as

$$\frac{P_c}{A_g} = \frac{\pi^2 \alpha h^2}{(1+\beta_d)(k\ell_u)^2} \tag{10}$$

To further generalize the expression, the slenderness ratio, $k\ell_u/r$, is introduced. r, the radius of gyration of the gross cross section may be expressed as $C_r h$, in which C_r is a characteristic coefficient based on the shape of the gross cross section. For rectangular and circular cross sections, C_r is $\sqrt{1/12} \approx 0.2887$ and 1/4, respectively. ACI 318-77 allows the former coefficient to be approximated as 0.30.

The critical stress may now be written as

$$\frac{P_c}{A_g} = \frac{\pi^2 \alpha}{(1+\beta_d)C_r^2\left(\frac{k\ell_u}{r}\right)^2} \tag{11a}$$

or

$$\frac{P_c}{A_g} = \frac{\pi^2 \alpha}{C_r^2 \lambda^2} \tag{11b}$$

in which

$$\lambda = \sqrt{1+\beta_d}\,\frac{k\ell_u}{r}$$

and is called the "modified slenderness ratio." For a given cross-sectional shape, reinforcing ratio, ϱ, and modified slenderness ratio, λ, critical stress, P_c/A_g, is constant.

The moment magnifier,

$$\delta = \frac{C_m}{1-(P_u/\phi P_c)} \geq 1.0$$

may be expressed in terms of the "factored stress," P_u/A_g, and the critical stress.

$$\delta = \frac{C_m}{1-\left(\frac{P_u}{A_g}/\frac{\phi P_c}{A_g}\right)} \geq 1.0 \tag{12a}$$

Since the factored axial load, P_u, must be less than or equal to the design axial strength, ϕP_n, equating the factored load to the design strength ($P_u = \phi P_n$) will provide a conservative approximation for the moment magnifier, δ.

$$\delta = \frac{C_m}{1-\left(\frac{\phi P_n}{A_g}/\frac{\phi P_c}{A_g}\right)} \geq 1.0 \tag{12b}$$

This form of the expression for δ is consistent with the generalized column capacities.

The moment index of conventional interaction diagrams is written in terms of the design moment, ($\phi P_n e = \phi M_n$), divided by the product $A_g h$. Rewriting ACI Eq. (10-6) in similar terms gives

$$\frac{M_c}{A_g h} = \delta\,\frac{M_2}{A_g h} \tag{13}$$

in which M_c is the magnified factored column moment used for design ($M_c = M_u \leq \phi M_n$), and M_2 is the larger of the factored end moments obtained from the elastic frame analysis. Dividing both sides of Eq. (13) by the moment magnifier and substituting ϕM_n for M_c gives

$$\frac{M_2}{A_g h} = \frac{1}{\delta}\,\frac{\phi M_n}{A_g h} \tag{14}$$

By using the expression for the moment magnifier in Eq. (12b), Eq. (14) may be rewritten.

$$\frac{M_2}{A_g h} = \frac{1-\left(\frac{\phi P_n}{A_g}/\frac{\phi P_c}{A_g}\right)}{C_m}\,\frac{\phi M_n}{A_g h} \tag{15}$$

Since C_m varies with the particular column constraints and loading, it is more useful to incorporate C_m on the left side of the equation in a "slender column moment index," which is defined as

$$\frac{C_m M_2}{A_g h} = \left[1-\left(\frac{\phi P_n}{A_g}/\frac{\phi P_c}{A_g}\right)\right]\frac{\phi M_n}{A_g h} \tag{16}$$

To develop a slender column interaction diagram, the axial load and moment indices, $\phi P_n/A_g$ and $\phi M_n/A_g h$, are determined for various locations of the neutral axis for a particular column geometry and reinforcing ratio, in the normal manner.[4] The value of the moment index is then modified using Eq. (16) to obtain the slender column moment index. The locus of points represented by the axial load index and the slender column moment index represent the interaction diagram for the column. Since P_c/A_g is a function of the modified slenderness ratio, λ, as well as geometry and material properties, separate interaction diagrams must be constructed for each combination of λ, f_y, f_c', column shape and steel arrangement.

Fig. 2 is a typical interaction diagram without slenderness effects ($\lambda = 0$). This particular diagram may be used for rectangular columns with equal reinforcing on all four faces, $f_c' = 4$ ksi (27.6 MPa), $f_y = 60$ ksi (414 MPa), and $\gamma = 0.75$. Fig. 3 is a slender column interaction diagram for the same parameters with a modified slenderness ratio, λ, equal to 55. Additional interaction diagrams are presented in Reference 2 for the column configurations shown in Fig. 1, γ values of 0.60, 0.75, and 0.90, concrete strengths of 4, 5, and 6 ksi (27.6, 34.5, and 41.4 MPa), Grade 60 (414 MPa) reinforcing steel, and 12 modified slenderness ratios, λ, ranging from 0 to 100 (540 interaction diagrams in all).

To provide reasonably complete coverage for a practical set of design charts, the values of λ should be selected so that the consecutive values of λ^2 are separated by approximately equal increments (the critical stress is inversely proportional to λ^2) of moderate size (about 1000). The specific values of λ used in Reference 2 (0, 30, 45, 55, 65, 70, 75, 80, 85, 90, 95, 100) were selected to meet these criteria. Linear interpolation between charts with different values of λ is conservative.[2]

Checking the Moment Magnifier

When C_m is less than 1, the moment magnifier, δ, can be less than 1. Therefore, the moment magnifier should be checked to prevent the design of an inadequately reinforced column. Rewriting Eq. (14), the moment magnifier is

$$\delta = \frac{\phi M_n}{A_g h} / \frac{M_2}{A_g h} \tag{17a}$$

or in terms of the slender column moment index, $C_m M_2/A_g h$

$$\delta = C_m \frac{M_n}{A_g h} / \frac{C_m M_2}{A_g h} \tag{17b}$$

Once the reinforcing ratio, ρ, has been determined from the appropriate slender column interaction diagram, the corresponding magnified moment index, $\phi M_n/A_g h$, may be back calculated by entering the interaction diagram for a short column (modified slenderness ratio, $\lambda = 0$) with the given axial index, $\phi P_n/A_g$, and reinforcing ratio. The moment magnifier is then checked using Eq. (17b).

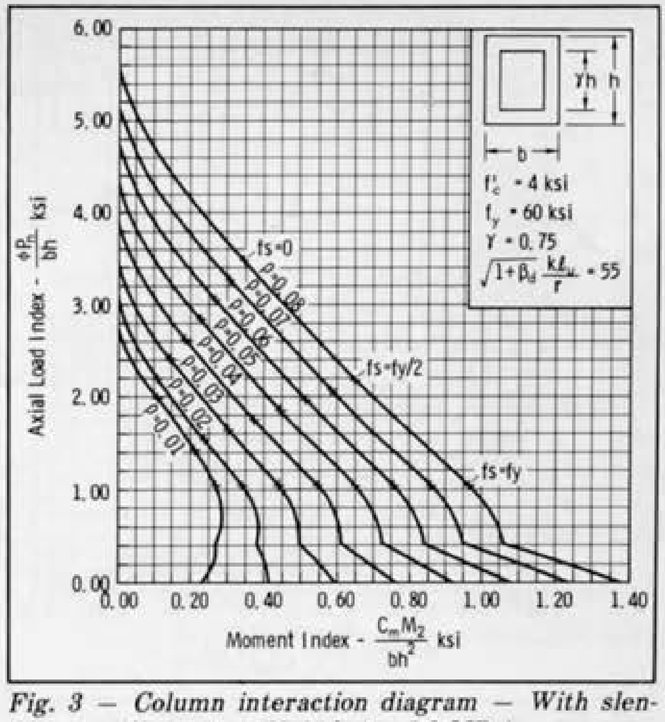

Fig. 3 — Column interaction diagram — With slenderness effects, $\lambda = 55$ (1 ksi = 6.9 MPa).

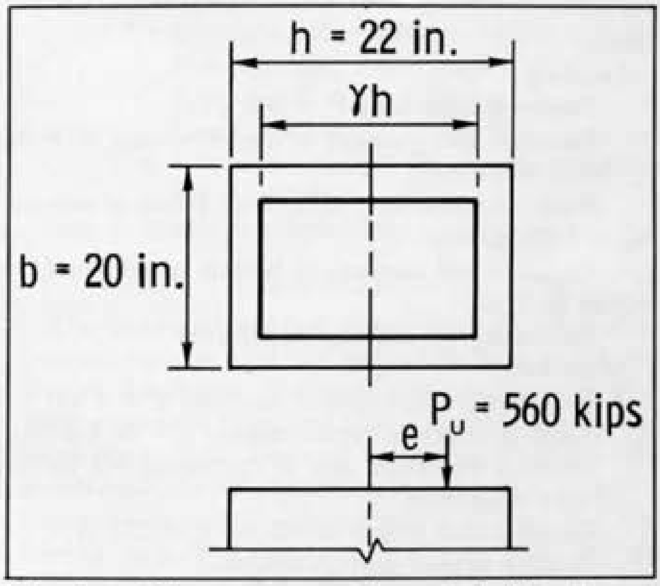

Fig. 4 — Column cross-section and loading for example problems (1 in. = 25.4 mm, 1 kip = 4.45 kN).

EXAMPLES

The two examples that follow compare the slender column interaction diagram approach to the procedure used in the ACI Design Handbook.[3] "Columns Example 2 — Selection of reinforcement for a rectangular tied column with bars on four faces (slenderness ratio found to be above critical value)" is used.

The problem is stated as follows (Note: 1 inch = 25.4 mm, 1 kip = 4.45 kN, 1 ksi = 6.9 MPa): For a 22 x 20 in. rectangular tied column with bars equally distributed along four faces (Fig. 4), select the reinforcement.

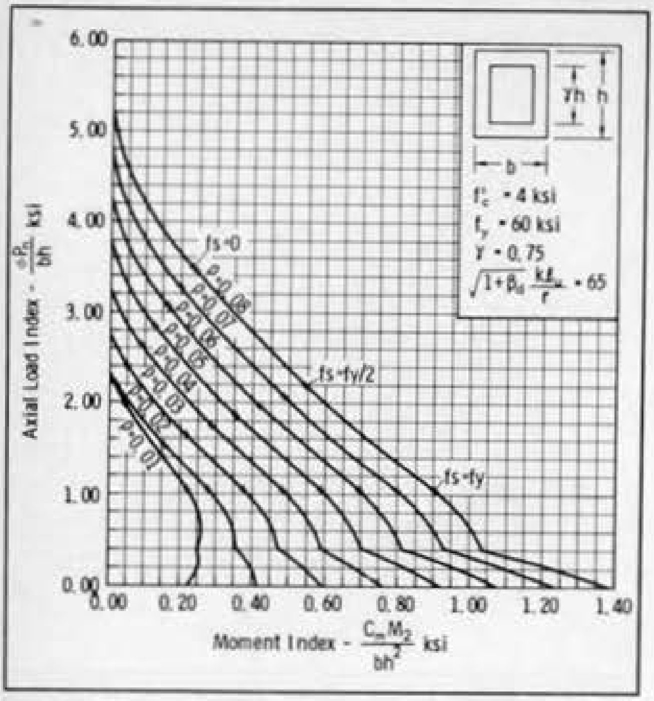

Fig. 5 — Column interaction diagram — With slenderness effects, λ = 65 (1 ksi = 6.9 MPa).

Given:

Loading

Factored axial load P_u = 560 kips

Factored end moment at top of column $M_u = M_2$ = +3920 in.-kips

Dead load moment, unfactored at top of column, M_d = 1120 in.-kips

Factored end moment at bottom of column M_1 = +2940 in.-kips

No transverse loading on member

Materials

Compressive strength of concrete f'_c = 4 ksi

Yield strength of reinforcement f_y = 60 ksi

Nominal maximum size of aggregate = 1 in.

Design conditions

Unsupported length of column ℓ_u = 27.5 ft

Column braced against sidesway

Slender Column Interaction Diagram Method[2]

Step 1. Determine column section size (same as Columns Example 2)

Given: h = 22 in.

b = 20 in.

Step 2. Check whether slenderness effects must be considered (same as Columns Example 2 or alternately)

a. Calculate the slenderness ratio*

For columns braced against sidesway, k = 1.0.

$$r = \sqrt{\frac{1}{12}}\,h = \sqrt{\frac{1}{12}}\,22 = 6.35 \text{ in.}$$

$$\frac{k\ell_u}{r} = \frac{1.0(27.5 \times 12)}{6.35} = 52.0$$

b. Compare $\frac{k\ell_u}{r}$ with 34-12 M_1/M_2

$$34\text{-}12\ M_1/M_2 = 34\text{-}12\ (2940/3920) = 25$$

$$\frac{k\ell_u}{r} > 34\text{-}12\ M_1/M_2$$

Slenderness effects must be considered.

Step 3. Determine reinforcing ratio and steel required.

a. Determine the axial load index and moment index

$$\frac{P_u}{bh} = \frac{560}{20(22)} = 1.27 \text{ ksi}$$

Since there is no transverse load on the column,

$$C_m = 0.6 + 0.4\ (M_1/M_2) =$$

$$0.6 + 0.4\left(\frac{2940}{3920}\right) = 0.9$$

$$\frac{C_m M_2}{bh} = \frac{(0.9)3920}{20(22)^2} = 0.364 \text{ ksi}$$

b. Determine appropriate interaction diagram(s).

$$\text{Estimate } \gamma \approx \frac{h\text{-}5}{h} = \frac{22\text{-}5}{22} = 0.77$$

Use $\gamma = 0.75$

Calculate the modified slenderness ratio

$$\beta_d = (1.4M_d)/M_u = \frac{1.4(1120)}{3920} = 0.4$$

$$\lambda = \sqrt{1+\beta_d}\,\frac{k\ell_u}{r} = \sqrt{1+0.4}\ 52.0 = 61.5$$

Interpolate between modified slenderness ratios of 55 and 65 for f'_c = 4 ksi, f_y = 60 ksi, γ = 0.75, and equal reinforcing in all faces.

c. From the interaction diagram for λ = 65 (Fig. 5), ϱ = 0.034.

From the interaction diagram for λ = 55 (Fig. 3), ϱ = 0.027

Interpolating for λ = 61.5

$$\varrho = 0.027 + (0.034\text{-}0.027)\,\frac{61.5\text{-}55}{65\text{-}55} = 0.032$$

*In Reference 2, the theoretical value of $\sqrt{1/12}\,h$ (=0.2887h) is used for the radius of gyration for rectangular columns rather than the value of 0.30h allowed by ACI 318-77.[1]

d. Since C_m is less than 1.0, check moment magnifier.

From the interaction diagram for $\lambda = 0$ (Fig. 2), with

$$\frac{\phi P_n}{bh} = \frac{P_u}{bh} \text{ at } \varrho = 0.032, \frac{\phi M_n}{bh^2} = 0.59$$

$$\delta = C_m \frac{\phi M_n}{bh^2} / \frac{C_m M_2}{bh^2} = 0.9(0.59/0.364)$$

$$= 1.46 > 1.0.$$

e. Compute steel required

$A_{st} = \varrho bh = 0.032(20)22 = 14.1$ in.^{2}

Step 4. Select reinforcement (same as Columns Example 2)

ACI Design Handbook Approach[3]

Step 1. Determine column section size

Given: $h = 22$ in.

$b = 20$ in.

Step 2. Check whether slenderness ratio is less than critical value

a. Compute M_1/M_2 and read critical value of $k\ell_u/h$ from design aid, Columns 1

$$M_1/M_2 = \frac{2940}{3920} = 0.75$$

Critical $k\ell_u/h = 7.5$

b. Determine k

For columns braced against sidesway, $k = 1.0$

c. Compute $k\ell_u/h$ and compare with critical value

$k\ell_u/h = 1.0(27.5)12/22 = 15 > 7.5$

Slenderness effects must be considered.

d. Determine moment magnification factor

Determine the appropriate moment magnifier graph.

For rectangular tied column with $f_c' = 4$ ksi, use design aid, Columns 5.2

Compute $\beta_d = (1.4M_D)/M_u = 1.4(1120)/3920 = 0.4$

Compute $P_u(1 + \beta_d)/A_g = 560(1.4)/(22 \times 20) = 1.78$ ksi

Since there is no transverse load, compute C_m from

$C_m = 0.6 + 0.4(M_1/M_2) = 0.6 + 0.4(2940/3920) = 0.9$

$$\text{Estimate } \gamma \approx \frac{h\text{-}5}{h} = \frac{22\text{-}5}{22} = 0.77 \approx 0.75$$

Determine appropriate interaction diagram.

For rectangular tied column with steel along four faces, $f_c' = 4$ ksi, $f_y = 60$ ksi, and $\gamma \approx 0.75$, use design aid, R4-60.75.

Assume trial values of ϱ	0.02	0.03	0.04
Read h_s/h	1.00	1.10	1.20
Compute $k\ell_u/h_s = k\ell_u/h \div h_s/h$	15.00	13.60	12.50
Read δ/C_m from graph	1.91	1.64	1.49
Compute $\delta = C_m(\delta/C_m)$	1.72	1.48	1.34
e. Compute δM_u, in.-kips	6740	5800	5250

Step 3. Determine reinforcing ratio and steel required.

a. Compute $P_u/A_g = 560/440 = 1.27$ ksi

Assumed ϱ	0.02	0.03	0.04
b. Compute $\delta M_u/A_g h$	0.696	0.599	0.542
c. Read ϱ for P_u/A_g and $\delta M_u/A_g h$	0.042	0.032	0.027
Compare with assumed ϱ	$\neq 0.02$	≈ 0.03	$\neq 0.04$

Repeat from Step 2d, assuming $\varrho = 0.033$

Read $h_s/h = 1.13$

Compute $k\ell_u/h_s = 13.3$

Read $\delta/C_m = 1.60$

Compute $\delta = 1.44$

Compute $\delta M_u = 5640$ in.-kips

Compute $\delta M_u/A_g h = 0.583$

Read $\varrho = 0.031 \approx 0.033$

Use $\varrho = 0.031$

d. Compute required $A_{st} = \varrho_g A_g = 0.031 \times 440 = 13.6$ in.^{2}

Step 4. Select reinforcement

DISCUSSION

The proposed method shows considerable computational savings over the method outlined in the ACI Design Handbook.[3] The Handbook procedure is an iterative process requiring initial assumptions for the reinforcing ratio, followed by successive trials. The slender column interaction method is a direct method, requiring no initial assumptions regarding the reinforcing ratio. The example calculations show that the proposed method provides results in a single solution which can be achieved only after several iterations using the method illustrated in the ACI Design Handbook.

SUMMARY AND CONCLUSIONS

A procedure for developing slender column interaction diagrams is presented. The provisions of ACI 318-77 for the approximate evaluation of slenderness effects are utilized.

Conventional interaction diagrams are modified using a nondimensional parameter characterizing column slenderness. The abscissa is modified to reflect the (unmagnified) moment obtained from an elastic frame analysis.

The interaction diagrams developed provide a direct solution for the reinforcing ratio of single columns. To use the diagrams, the designer does not have to make initial reinforcing ratio assumptions or iterative calculations. The approach presented here will reduce the design time for reinforced concrete structures.

ACKNOWLEDGEMENTS

The work reported in this paper was supported by the Marley Cooling Tower Company, Mission, Kansas.

NOTATION

A_g = gross area of column cross section

A_{st} = "total area of longitudinal reinforcement,"[1]

C_g = characteristic coefficient for calculation of the gross moment of inertia, I_g

C_m = "a factor relating actual moment diagram to an equivalent uniform moment diagram,"[1]

C_r = characteristic coefficient for calculation of the radius of gyration, r

C_{st} = characteristic coefficient for calculation of the reinforcing steel moment of inertia, I_{st}

e = eccentricity of axial column load

E_c = "modulus of elasticity of concrete,"[1]

E_s = "modulus of elasticity of reinforcement,"[1]

EI = "flexural stiffness of compression member,"[1]

f'_c = "specified compressive strength of concrete,"[1]

f_y = "specified yield strength of nonprestressed reinforcement,"[1]

h = dimension of the cross section in the direction perpendicular to the neutral axis of bending

I_g = "moment of inertia of gross concrete section,"[1]

I_{st} = "moment of inertia of reinforcement,"[1]

k = "effective length factor for compression members,"[1]

ℓ_u = "unsupported length of compression member,"[1]

M_c = "factored moment to be used for design of compression member,"[1]

M_u = factored moment

M_1 = "value of smaller factored end moment on compression member calculated by conventional elastic frame analysis, positive if member is bent in single curvature, negative if bent in double curvature,"[1]

M_2 = "value of larger factored end moment on compression member calculated by conventional elastic frame analysis, always positive,"[1]

P_c = "critical load,"[1]

P_n = "nominal axial load strength at given eccentricity,"[1]

P_u = "factored axial load at given eccentricity $\leq \phi P_n$,"[1]

r = radius of gyration

α = function combining terms of EI, Eq. (9)

β_d = "ratio of maximum factored dead load moment to maximum factored total load moment, always positive,"[1]

γ = geometric index, ratio of the distance between the centroids of the reinforcement in opposite faces parallel to the neutral axis of bending to the column depth, h, as shown in Fig. 1

δ = "moment magnification factor,"[1] or moment magnifier

λ = modified slenderness ratio = $\sqrt{1+\beta_d}\,(k\ell_u/r)$

ϱ = total reinforcing ratio = A_{st}/A_g

ϕ = "strength reduction factor,"[1]

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Received and reviewed under Institute publication policies.

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